Fourier Series - Definition

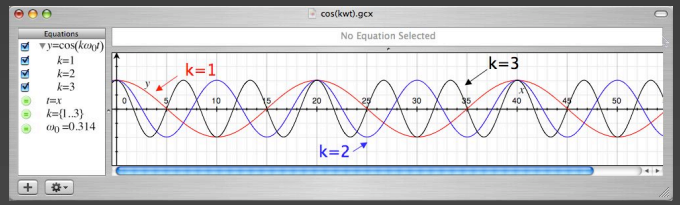
$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{jk\omega_0 t}$$

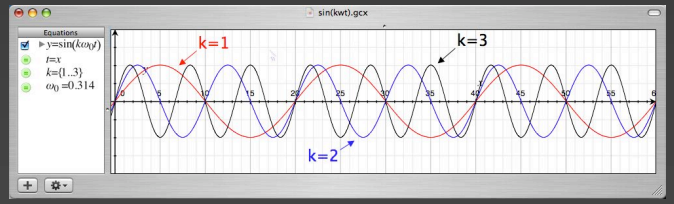
$$\omega_0 = 2 \cdot \pi \cdot f_0 = \frac{2 \cdot \pi}{T_0}$$

$k\omega_0 = Frequency$

$\omega_0 = Fundamental\ Frequency$

$$e^{jk\omega_0 t} = cos(k\omega_0 t) + j\, sin(k\omega_0 t)$$





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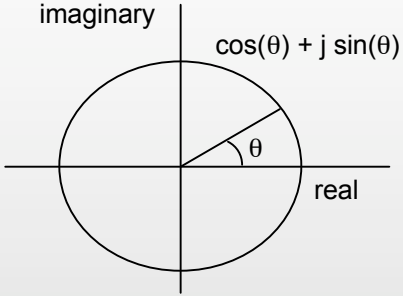
Euler : Complex Exponentials

$$e^{j\theta} = cos(\theta) + j\ sin(\theta)$$

$$Magnitude(e^{j\theta}) = 1$$

$$Phase(e^{j\theta}) = \theta$$



$$\frac{d}{dx}\cos(\theta) = -\ sin(\theta) \qquad \frac{d}{dx}sin(\theta) = \cos(\theta)$$

Side Note :

$$\frac{d}{dx}(e^{j\theta}) = j\ e^{j\theta}$$

$$\cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

$$\sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

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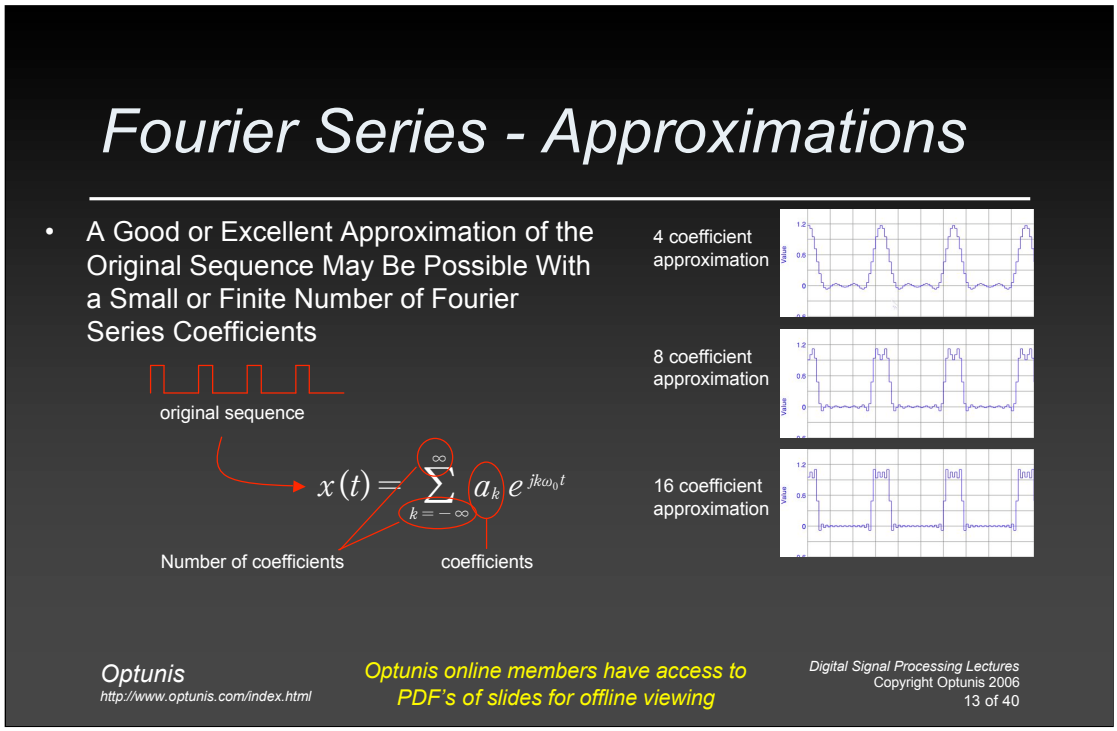

For many signals, most of the weights quickly drop off towards zero.

In this case, we may approximate the signal with a minimum of terms. Naturally, the more terms we have, the better the approximation.

Some Fourier Transform Properties

- Linearity $a \cdot x_1(t) + b \cdot x_2(t) \Leftrightarrow a \cdot X_1(\omega) + b \cdot X_2(\omega)$
- Symmetry $\mathrm{Re}\,al\{x(t)\} \Leftrightarrow X(-\omega) = X^*(\omega)$
- Time Shifting $x(t - t_0) \Leftrightarrow e^{-j\omega t_0} X(\omega)$
- Frequency Shifting $e^{j\omega_0 t} x(t) \Leftrightarrow X(\omega - \omega_0)$
- Convolution $\int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau \Leftrightarrow X(\omega)\, H(\omega)$

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The Fourier transform has several interesting properties.

Linearity means that if we can add two signals in time, their Fourier transforms will also add. If we amplify or attenuate a signal in time, its Fourier transform will also amplify or attenuate by the same amount.

If a signal is real in time, it will be complex conjugate symmetric. This means, that we only really need to look at half the Fourier transform.

Time shifting, will be described in detail in a later slide.

Shifting the frequency of a signal is the same as multiplying the signal in time by a rotating complex exponential

Finally, multiplying two signal in the frequency domain is the same as complicated process called convolution in the time domain.